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Binding Energies of Nuclei

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A new theoretical method is proposed to describe known properties of nuclei. The method is based on the theory of nuclear forces given in an earlier paper and results in formulas for the binding energies and dimensions of nuclei which accord with experimental data.

Let us consider a classical particle which interacts with nuclear and electromagnetic fields. Its movement in the fields can be described by the following equations (Rabinowitch, 1994):

$$\rho_0 \exp(\varphi/c^2)\,(c^2\, d^2x^k/ds^2 + d\varphi/ds\; dk^k/ds - \partial\varphi/\partial x_k) - \theta F^k_n\, dx^n/ds = 0 \quad (1)$$

$$(x^1, x^2, x^3) \in \Omega(x^0) \quad (2)$$

$$\partial^2\varphi/\partial x^n\, \partial x_n + (m_\pi c/\hbar)^2\varphi = -4\pi(G/m_p)^2\rho_0 \exp(\varphi/c^2)$$

$$F_{kn} = \partial A_n/\partial x^k - \partial A_k/\partial x^n \quad (3)$$

$$\partial^2 A^k/\partial x^n \partial x_n = 4\pi\theta\, dx^k/ds, \qquad \partial A^k/\partial x^k = 0$$

where φ is the scalar potential of nuclear forces, A_k are electromagnetic potentials, ρ_0 is the density of the particle mass m_0 at rest when $\varphi = 0$, θ is the density of the particle charge, dx^k/ds is the 4-vector of its velocity, $ds^2 = dx^k dx_k$, m_π and m_p are the masses at rest of the neutral pion and proton, respectively, $G^2/\hbar c$ is a dimensionless constant of the strong interaction, and $\Omega(x^0)$ is the small spatial volume, depending on time, which is occupied by the moving particle.

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Let us write down a condition which must be imposed on the mass of the particle to regard it as classical and hence for equation (1) to be correct. The condition follows from the known correlation (Naumov, 1984)

$$(2mE_b)^{1/2}r_* >> \hbar \tag{4}$$

where E_b, m, and r_* are the binding energy, mass at rest, and radius of the particle, respectively.

Since $r \sim N^{1/3}$ fm, $E_b/N \sim 10^{-2}\, m_p c^2$ (Naumov, 1984), where $N = m/m_p$, from (4) we get

$$(m/m_p)^{4/3} >> 1 \tag{5}$$

Therefore, we can consider a particle classical when condition (5) is satisfied. In accordance with (5), we will further examine nuclei with $m/m_p \geq 20$.

Let us consider a nonrelativistic nucleus moving under the action of external sources of the electromagnetic field with potentials A_n^{ext} and apply equations (1)–(3) to it. Then in the nonrelativistic case under consideration we get

$$\rho_0 \exp(\varphi/c^2)\,(w^\alpha + \partial\varphi/\partial x^\alpha) + \theta(\partial A_0/\partial x^\alpha - \partial A_\alpha/\partial x^0) = 0, \qquad \alpha = 1, 2, 3 \tag{6}$$

$$(x^1, x^2, x^3) \in \Omega(x^0), \qquad A_n = A_n^{\text{int}} + A_n^{\text{ext}} \tag{7}$$

$$\Delta A_0^{\text{int}} = -4\pi\theta, \qquad \Delta A_\alpha^{\text{int}} = 4\pi\theta v^\alpha/c \tag{8}$$

where $v^\alpha = v^\alpha(x^0)$ and $w^\alpha = w^\alpha(x^0)$ are the velocity and acceleration, respectively, of the nucleus, and A_n^{int} are the potentials of the electromagnetic field generated by the nucleus itself.

From (8) we obtain the correlation for A_n^{int}

$$A_\alpha^{\text{int}} = -A_0^{\text{int}} v^\alpha/c, \qquad v^\alpha = v^\alpha(x^0), \qquad \alpha = 1, 2,3 \tag{9}$$

$$A_0^{\text{int}} = \int_\Omega \theta/R\, dy^1\, dy^2\, dy^3, \qquad R = [(x^1 - y^1)^2 + (x^2 - y^2)^2 + (x^3 - y^3)^2]^{1/2} \tag{10}$$

Therefore, from (6), (7), and (9) we derive the following equation in the considered case $|v^\alpha/c| << 1$:

$$[c^2 \exp(\varphi/c^2) + \gamma A_0^{\text{int}}]w^\alpha/c^2 + \partial[c^2 \exp(\varphi/c^2)]/\partial x^\alpha + \gamma\, \partial A_0^{\text{int}}/\partial x^\alpha = \gamma E_{\text{ext}}^\alpha$$

$$\gamma = \theta/\rho_0, \qquad \alpha = 1, 2, 3 \tag{11}$$

where E_{ext}^α are electric field intensities generated by the sources external to the nucleus.

Let us assume that the external sources are remote from the nucleus. Then inside the particle the external electromagnetic field is homogeneous and $\varphi = \varphi^{\text{int}}$, where φ^{int} is the potential of the nuclear field generated by the nucleus itself.

Since the accelerations w^α must be the same for different points of the particle and E^α_{ext} are arbitrary but also the same for those points, from (11) we derive the two equations

$$\gamma = \theta/\rho_0 = \text{const}$$

$$c^2 \exp(\varphi^{\text{int}}/c^2) + \gamma A_0^{\text{int}} = \text{const}, \qquad (x^1, x^2, x^3) \in \Omega(x^0) \tag{12}$$

Equations (11) and (12) give the classical law of Newtonian mechanics

$$mw^\alpha = qE^\alpha_{\text{ext}} \tag{13}$$

where q is the charge of the nucleus and m is its inertial mass, given by the following expression:

$$m = m_0 \exp(\varphi^{\text{int}}/c^1) + qA_0^{\text{int}}/c^2 = \text{const}$$

$$m_0 = q\rho_0/\theta = \text{const}, \qquad (x^1, x^2, x^3) \in \Omega(x^0) \tag{14}$$

The two equations (14) permit us to describe the distributions in the nucleus of the charge q with the density θ and the mass m_0 with the density ρ_0. As follows from (1) and (14), m_0 is the nucleus mass at rest when $\varphi = 0$.

Let us now examine a nucleus which is at rest relative to an inertial frame of reference. In this case $E^\alpha_{\text{ext}} = 0$. We have a stationary, spherically symmetric problem. Let r and r_* be the distance between a point of the nucleus and its center and the radius of the nucleus, respectively. Then we have $r \leq r_*$.

For the region $r \leq r_*$ occupied by the nucleus we apply equations (2), (8), and (14) and get

$$f'' + 2f'/r - (m_\pi c/\hbar)^2 f = 4\pi\theta(m_0/q)[G/(m_p c)]^2 e^f, \qquad f = \varphi^{\text{int}}(r)/c^2 \tag{15}$$

$$\beta'' + 2\beta'/r = -4\pi\theta/c^2, \qquad \beta = A_0^{\text{int}}(r)/c^2 \tag{16}$$

$$m_0 e^f f' + q\beta' = 0, \qquad 0 \leq r \leq r_* \tag{17}$$

From (16) and (17) we find

$$\beta' = -(m_0/q)e^f f', \qquad \beta'' = -(m_0/q)e^f[f'' + (f')^2] \tag{18}$$

$$4\pi\theta/c^2 = (m_0/q)e^f[f'' + 2f'/r + (f')^2], \qquad f = f(r) \tag{19}$$

From (15) and (19) we derive the following equation for the nuclear potential φ^{int}:

$$(\mu^2 e^{2f} - 1)(f'' + 2f'/r) + \mu^2 e^{2f}(f')^2 + \nu^2 f = 0, \qquad 0 \leq r \leq r_* \tag{20}$$

where

$$\mu = Gm_0/m_pq, \qquad \nu = m_\pi c/\hbar \tag{21}$$

Let us consider equation (2) for φ^{int} in the region $r > r_*$ outside the nucleus. This equation has the following form:

$$f'' + 2f'/r - \nu^2 f = 4\pi[G/(m_pc)]^2\epsilon_0 e^f, \qquad f = \varphi^{\text{int}}/c^2, \qquad r > r_* \tag{22}$$

where $\epsilon_0 = \epsilon_0(r)$ is the density of the mass at rest of virtual pions (Feynman, 1961) created in the physical vacuum at the nucleus surface $r = r_*$ because of the influence on it of this surface.

Considering that the lifetime of the virtual pion is given by $\tau \sim \hbar/m_\pi c^2$ (Naumov, 1984) and their mean speed is $|\bar{v}| << c$ as the surface $r = r_*$ is immovable, we find that the mass of the virtual pions is mainly concentrated in a very narrow region, $r_* < r < r_* + \Delta$, where $\Delta << \hbar/m_\pi c$.

Therefore, the mass density $\epsilon_0(r)$ of the virtual pions can approximately be represented by means of the delta function $\delta(r)$ as

$$\epsilon_0(r) = \sigma\delta(r - r_*), \qquad r > r_*, \qquad \text{where} \int_0^\infty \delta(r)\, dr = 1 \tag{23}$$

σ is a constant equal to the mass of the virtual pions per unit area of the nucleus surface $r = r_*$.

Let us define

$$x = \nu r, \qquad x_* = \nu r_*, \qquad \nu = m_\pi c/\hbar \tag{24}$$

Then from (20) and (22) we get

$$(\mu^2 e^{2f} - 1)(f'' + 2f'/x) + \mu^2 e^{2f}(f')^2 + f = 0, \qquad 0 \le x \le x_* \tag{25}$$

$$f'' + 2f'/x - f = \lambda_0 e^f, \qquad x > x_* \tag{26}$$

$$\lambda_0 = \lambda_0(x) = 4\pi\epsilon_0(x)[G/(m_pc\nu)]^2, \qquad f = f(x), \qquad f(\infty) = 0 \tag{27}$$

From equation (26) and the condition $f(\infty) = 0$ we obtain the integral equation

$$f(x) = \left[e^x \int_\infty^x \lambda_0 e^{f-x} x\, dx - e^{-x}\left(\int_{x_*}^x \lambda_0 e^{f+x} x\, dx + D\right)\right] \Big/ (2x) \tag{28}$$

$$D = \text{const}, \qquad x \ge x_*$$

Equation (28) gives the following formulas for $f(x_*)$ and $f'(x_*)$:

$$f(x_*) = -\left[\int_{x_*}^{\infty} \lambda_0 e^{f-x} x \, dx \, e^{x_*} + De^{-x_*}\right] \Big/ (2x_*)$$

$$f'(x_*) = \left[\int_{x_*}^{\infty} \lambda_0 e^{f-x} x \, dx \, (1 - x_*)e^{x_*} + D(1 + x_*)e^{-x_*}\right] \Big/ (2x_*^2) \quad (29)$$

From (29) we find

$$f(x_*)(1 + x_*) + x_* f'(x_*) = -e^{x_*} \int_{x_*}^{\infty} \lambda_0 e^{f-x} x \, dx \quad (30)$$

Correlation (30) is a condition at the point $x = x_*$ for the solution of equations (25) and (26) to vanish at infinity.

From (23), (27), and (30) we get the equality

$$f(x_*)(1 + x_*) + x_* f'(x_*) = -s x_* e^{f(x_*)} \sigma \quad (31)$$

where

$$s = 4\pi[G/(m_p c)]^2/\nu, \qquad \sigma = \text{const} \quad (32)$$

Let us turn to the electromagnetic potential A_0^{int}, for which we have the classical formula in the region $r \geq r_*$

$$\beta(r) = A_0^{\text{int}}(r)/c^2 = q/c^2 r, \qquad r \geq r_* \quad (33)$$

q is the nucleus charge.

From equations (16), (24), and (33) we obtain

$$\beta(x) = \beta(0) + 4\pi \int_0^x t(t/x - 1)\theta(t) \, dt/(c\nu)^2, \qquad 0 \leq x \leq x_* \quad (34)$$

$$\beta(x) = q\nu/c^2 x, \qquad x \geq x_* \quad (35)$$

From (34) and (35) we get

$$\beta'(x_*) = -4\pi \int_0^{x_*} t^2 \theta(t) \, dt/(c\nu x_*)^2 = -q\nu/(cx_*)^2 \quad (36)$$

Formulas (19) and (36) give

$$\int_0^{x_*} x^2 e^f [f'' + 2f'/x + (f')^2] \, dx = q^2 \nu / m_0 c^2, \qquad f = f(x) \quad (37)$$

Let us now examine equation (25). From it we get

$$y'' + 2y'/x = -H(x), \qquad 0 \leq x \leq x_* \quad (38)$$

where

$$y = e^{f(x)}, \qquad H(x) = [y^2 \ln(y) + (y')^2]/[y(\mu^2 y^2 - 1)] \tag{39}$$

From (38) we obtain the integral equation

$$y(x) = y(0) + \int_0^x t(t/x - 1)H(t)\,dt \tag{40}$$

Equation (40) can be represented in the form

$$y(x) = u/x + v, \qquad u = \int_0^x t^2 H(t)\,dt, \qquad v = y(0) - \int_0^x tH(t)\,dt \tag{41}$$

From (41) we easily obtain

$$y'(x) = -u/x^2 \tag{42}$$

Consider equality (37). It can be represented as follows:

$$\int_0^{x_*} x^2(y'' + 2y'/x)\,dx = q^2 v/m_0 c^2, \qquad y = e^{f(x)} \tag{43}$$

From (38) and (43) we get

$$\int_0^{x_*} x^2 H(x)\,dx = -q^2 v/m_0 c^2 \tag{44}$$

and, taking into account (41), we have

$$u(x_*) = -q^2 v/m_0 c^2 \tag{45}$$

Formulas (39), (42), and (45) give the following condition:

$$x_*^2 y'(x_*) = x_*^2 e^{f(x_*)} f'(x_*) = q^2 v/m_0 c^2 \tag{46}$$

From (31) and (46) we get one more condition at the point x_*,

$$f(x_*)(1 + x_*) + q^2 v e^{-f(x_*)}/m_0 c^2 x_* = -b x_* e^{f(x_*)}, \qquad b = s\sigma \tag{47}$$

By using (14), (33), and (35) we obtain the formula for the nucleus inertial mass m

$$m = m_0 e^{f(x_*)} + q\beta(x_*) = m_0 e^{f(x_*)} + q^2 v/c^2 x_* \tag{48}$$

Let A and Z be, respectively, the number of nucleons and the number of protons in a nucleus. Then, since m_0 is the nucleus mass at rest when $\varphi = 0$, we have

$$m_0 = Zm_0^p + (A - Z)m_0^n, \qquad q = Ze_p \tag{49}$$

where m_0^p and m_0^n are the masses at rest when $\varphi = 0$ of the proton and neutron, respectively, and e_p is the proton charge.

From (48) and (49) we get the formula for the binding energy E_b of the nucleus

$$\begin{aligned} E_b &= c^2[Zm_p + (A - Z)m_n - m] \\ &= c^2\{(A - Z)(m_n - m_p) + Zm_p[1 - (1 + \delta_p)e^{f(x_*)}] \\ &\quad + (A - Z)m_p[1 - (1 + \delta_n)e^{f(x_*)}]\} - Z^2e_p^2\nu/x_* \end{aligned} \tag{50}$$

where m_p and m_n are the experimental values of the masses at rest of the proton and neutron, respectively, and

$$m_0^p = m_p(1 + \delta_p), \qquad m_0^n = m_p(1 + \delta_n) \tag{51}$$

δ_p and δ_n are some constants.

Let us consider equation (25) with conditions (46) and (47) to determine x_* and $f(x_*)$. We seek their solution $f(x)$ in the form

$$f(x) = f_0\left(1 + \sum_{n=1}^{\infty} d_n x^{2n}\right), \qquad 0 \le x \le x_* \tag{52}$$

The function $e^{2f(x)}$ in (25) can be expressed as the power series

$$e^{2f(x)} = e^{2f_0} \sum_{k=0}^{\infty} 2^k f_0^k \left(\sum_{n=1}^{\infty} d_n x^{2n}\right)^k / k! = e^{2f_0} \sum_{n=0}^{\infty} p_n x^{2n} \tag{53}$$

$$p_0 = 1, \qquad p_n = \sum_{i=1}^{n} 2^i f_0^i q_{n,i}/i!, \quad n \ge 1, \qquad \sum_{n=i}^{\infty} q_{n,i} x^{2n} = \left(\sum_{n=1}^{\infty} d_n x^{2n}\right)^i$$

$$q_{n,i} = \sum_{j=i-1}^{n-1} d_{n-j} q_{j,i-1}, \qquad 1 \le i \le n, \qquad q_{0,0} = 1; \qquad q_{j,0} = 0, \quad j > 0 \tag{54}$$

From (25) and (53) we obtain

$$\begin{aligned} &2\left(D^2 \sum_{n=0}^{\infty} p_n x^{2n} - 1\right) \sum_{n=0}^{\infty} (n + 1)(2n + 3)d_{n+1}x^{2n} + \sum_{n=0}^{\infty} d_n x^{2n} \\ &+ 4D^2 f_0 \sum_{n=0}^{\infty} p_n x^{2n} \sum_{n=0}^{\infty} \sum_{k=0}^{n} (k + 1)(n - k + 1)d_{k+1}d_{n-k+1}x^{2n+2} \\ &= \sum_{n=0}^{\infty} H_n x^{2n} = 0 \end{aligned}$$

$$p_0 = 1, \quad d_0 = 1, \quad D = \mu e^{f_0},$$
$$H_n = H_n(D, f_0, d_1, d_2, \ldots, d_{n+1}), \quad 0 \le x \le x_* \tag{55}$$

From (55) we have the equations $H_n = 0$, $n \geq 0$, which give the recurrence correlations

$$\begin{aligned} d_{n+1} &= F(D, f_0, d_0, d_1, \ldots, d_n) \\ &= -\left\{ d_n + 2D^2 \sum_{i=0}^{n-1} [(i + 1)d_{i+1}(p_{n-i}(2i + 3) + 2f_0(n - i)d_{n-i}) \right. \\ &\quad \left. + 2f_0 p_{n-i} \sum_{k=0}^{i-1} (k + 1)(i - k)d_{k+1}d_{i-k}] \right\} \\ &\quad \times [(2n + 2)(2n + 3)(D^2 - 1)]^{-1}, \quad n \geq 0, \quad d_0 = 1, \quad \sum_0^{-1} = 0 \end{aligned} \tag{56}$$

From (56) we find

$$d_1 = -1/[6(D^2 - 1)], \quad d_2 = 0.3(1 + 16D^2 f_0 d_1)d_1^2$$
$$d_3 = [d_2 + 4D^2 f_0 d_1(5f_0 d_1^2 + 17d_2)]d_1/7, \ldots \tag{57}$$

where, owing to (21) and (49),

$$D^2 = \mu^2 e^{2f_0} = a(1 + \delta)^2 e^{2f_0} A^2/Z^2$$
$$a = G^2/e_p^2, \quad \delta = \delta_p Z/A + \delta_n(1 - Z/A) \tag{58}$$

Conditions (46) and (47) give the following equations:

$$RZ^2/A = 2(1 + \delta)x_*^3 f_0 e^{f(x_*)}(d_1 + 2d_2 x_*^2 + 3d_3 x_*^4 + \cdots)$$
$$R = (e_p^2/\hbar c)m_\pi/m_p = 0.00104966$$
$$f_0 = f(x_*)/(1 + d_1 x_*^2 + d_2 x_*^4 + d_3 x_*^6 + \cdots) \tag{59}$$
$$f(x_*)(1 + x_*) + Pe^{-f(x_*)} = -bx_* e^{f(x_*)}, \quad P = RZ^2/Ax_*(1 + \delta) \tag{60}$$

From (60) we can compute $f(x_*)$ and then from (56)–(59) we can determine $d_1, d_2, d_3, \ldots, f_0$, and x_* as functions of A, Z, and the four unknown dimensionless parameters a, b, δ_p, δ_n. Then from (50) we can determine the binding energy E_b of a nucleus as a function of these four parameters.

In order to determine the parameters a, b, δ_p, and δ_n we have performed computer calculations of $E_b(A, Z, a, b, \delta_p, \delta_n)$ by using (50) and (56)–(60) and compared them with the well-known experimental values $E_b^{\text{exp}}(A, Z)$ of the binding energies of nuclei (Acosta *et al.*, 1973). The parameters a and b

were varied over wide limits and δ_p and δ_n were determined as functions $\delta_p(a, b)$ and $\delta_n(a, b)$ from using the equation $E_b = E_b^{exp}$ twice for the two nuclei ${}^{107}_{47}Ag$ and ${}^{197}_{79}Au$ by the method of successive approximations.

In accordance with (5), we considered nuclei with $A \geq 20$. The comparison of the calculated binding energies E_b with E_b^{exp} gave the following values of the parameters a, b, δ_p, and δ_n:

$$a = 11.0, \qquad b = 0.049, \qquad \delta_p = 0.015608, \qquad \delta_n = 0.034606 \tag{61}$$

The results of the computer calculations are given in Table I. It is seen from Table I that the computed values E_b of the binding energies of nuclei are very close to their experimental values E_b^{exp}.

The values of the computed radii r_* of nuclei are also close to their experimental values. As follows from Table I, the radius r_* of a nucleus can be represented in the form

Table I. Computed Radii (r_*, fm) and Computed and Experimental Binding Energies of Nuclei per Nucleon (E_b/A and E_b^{exp}/A, MeV)

Z	A	r_*	E_b/A	E_b^{exp}/A	Z	A	r_*	E_b/A	E_b^{exp}/A
10	20	3.837	8.130	8.032	58	140	6.981	8.360	8.378
12	25	4.108	8.254	8.224	60	145	7.057	8.318	8.312
14	30	4.345	8.363	8.521	62	150	7.131	8.274	8.263
16	35	4.556	8.455	8.538	64	155	7.203	8.230	8.213
18	40	4.747	8.528	8.596	66	160	7.274	8.184	8.186
20	45	4.922	8.586	8.630	68	166	7.359	8.147	8.141
22	50	5.084	8.631	8.756	70	170	7.411	8.090	8.106
24	55	5.235	8.663	8.728	72	176	7.493	8.052	8.060
26	58	5.316	8.736	8.792	74	180	7.543	7.993	8.024
28	65	5.511	8.700	8.737	76	185	7.607	7.943	7.982
30	70	5.639	8.706	8.730	78	190	7.669	7.892	7.947
32	75	5.760	8.706	8.696	79	195	7.735	7.894	7.921
34	80	5.875	8.700	8.711	80	200	7.800	7.896	7.906
36	85	5.986	8.689	8.700	82	205	7.860	7.846	7.874
38	90	6.092	8.673	8.700	84	210	7.918	7.795	7.834
40	95	6.195	8.654	8.645	86	215	7.975	7.744	7.764
42	100	6.294	8.631	8.604	88	220	8.032	7.692	7.712
44	105	6.389	8.606	8.566	90	225	8.087	7.640	7.660
46	110	6.481	8.577	8.553	92	230	8.142	7.587	7.621
48	115	6.570	8.546	8.509	94	235	8.196	7.533	7.579
50	120	6.657	8.512	8.505	96	240	8.249	7.480	7.543
52	125	6.742	8.477	8.458	98	245	8.301	7.426	7.500
54	130	6.824	8.439	8.438	100	250	8.353	7.371	7.462
56	135	6.903	8.400	8.398	102	255	8.404	7.317	7.430

$$r_* = \alpha_*(A, Z)A^{1/3}, \qquad 1.325 < \alpha_*(A, Z) < 1.414 \text{ fm}, \qquad A \geq 20 \tag{62}$$

Formula (62) accords with experimental results on the interactions of nuclei with neutrons. It follows from these experiments that the radii r_* of nuclei are approximately proportional to $A^{1/3}$ and the coefficient $\alpha_*(A, Z) =$ 1.3–1.4 fm (Naumov, 1984).

Consider the nuclear potential φ of a nucleus. In the region $0 \leq r \leq r_*$ the function φ can be calculated by formulas (52), (56), and (60). Computer calculations showed that the power series in (52) has a very good convergence and we can approximately write

$$\varphi(r) = \varphi(0)[1 + d_1(\nu r)^2 + d_2(\nu r)^4 + d_3(\nu r)^6], \qquad 0 \leq r \leq r_* \tag{63}$$

where d_1, d_2, and d_3 are determined by formulas (57).

In the region $r \geq r_*$ the potential φ is as follows:

$$\varphi(r) = \varphi(r_*) \exp[\nu(r_* - r)]r_*/r, \qquad r \geq r_* \tag{64}$$

Let us consider the constant of strong interaction. As follows from (58) and (61), we have

$$G^2/\hbar c) \cong 0.080 \tag{65}$$

The value (65) is the known constant of the strong interaction of nucleons inside nuclei (Acosta *et al.*, 1973; Ericson and Weise, 1988). In this case we have low-energy interactions of nucleons.

Let us examine the case of high-energy interaction. Consider a nucleus which interacts with a high-energy particle. Let us choose an arbitrary instant of time x^0 and an inertial frame of reference in which at this instant the nucleus velocity is zero: $dx^\alpha/dx^0 = 0$, $\alpha = 1, 2, 3$.

Then for the nucleus at the chosen instant we have, from (1)–(3),

$$\begin{aligned} e^{\varphi/c^2}\, \partial\varphi/\partial x^k + \gamma(\partial A_0/\partial x^k - \partial A_k/\partial x^0) \\ = W_k, \qquad (x^1, x^2, x^3) \in \Omega(x^0) \end{aligned} \tag{66}$$

where

$$\gamma = \theta/\rho_0, \qquad W_k = e^{\varphi/c^2}(c^2\, d^2x_k/dx_0^2 + \partial\varphi/\partial x_0\, dx_k/dx_0) \tag{67}$$

$$\partial^2\varphi/\partial x^n \partial x_n + \nu^2\varphi = -4\pi(G/m_p)^2\rho_0 e^{\varphi/c^2}, \qquad \nu = m_\pi c/\hbar \tag{68}$$

$$\partial^2 A_0/\partial x^n \partial x_n = 4\pi\theta, \qquad \partial A_k/\partial x_k = 0 \tag{69}$$

$\Omega(x^0)$ is the spatial volume occupied by the nucleus.

From (66) we get

$$\partial W_k/\partial x_k = e^{\varphi/c^2}\, \partial^2\varphi/\partial x^k \partial x_k + \gamma(\partial^2 A_0/\partial x^k \partial x_k - \partial^2 A_k/\partial x_k \partial x^0) + U \tag{70}$$

where

$$U = (1/c^2)e^{\varphi/c^2}\, \partial\varphi/\partial x^k\, \partial\varphi/\partial x_k + \partial\gamma/\partial x_k\, (\partial A_0/\partial x^k - \partial A_k/\partial x^0) \tag{71}$$

From (68)–(70) we find

$$U - \partial W_k/\partial x_k = e^{\varphi/c^2}[4\pi(G/m_p)^2\rho_0 e^{\varphi/c^2} + \nu^2\varphi] - 4\pi\theta\gamma, \qquad \gamma = \theta/\rho_0 \tag{72}$$

Hence

$$4\pi\theta\gamma[(G/m_p\gamma)^2 e^{2\varphi/c^2} - 1] = U - \partial W_k/\partial x_k - \nu^2\varphi e^{\varphi/c^2} \tag{73}$$

As follows from (73), inside the nucleus the extreme value of the nuclear potential φ satisfies the correlation

$$Ge^{\varphi/c^2}/m_p\gamma = 1 \tag{74}$$

since in this case we have the infinite value of the charge density: $\theta = \infty$.

It must be noted that formula (74) is also correct when quantum mechanical effects are essential ($A < 20$). In this case only formula (67) for W_k must be changed and therefore the left-hand side of (73) and hence formula (74) are right.

For the proton, $\gamma = \text{const} \cong e_p/m_p$ and from (74) we find the extreme value of the nuclear potential inside the proton

$$\varphi = -c^2 \ln(G/e_p) \tag{75}$$

Outside the proton we have

$$m_p\varphi = -g^2 e^{-\nu r}/r \tag{76}$$

where $g^2/\hbar c$ is a dimensionless characteristic of the strong interaction.

It follows from (75) and (76) that the extreme value of g can be determined by the correlation

$$g^2 e^{-\nu r_p}/r_p = -m_p\varphi(r_p) = m_p c^2 \ln(G/e_p) \tag{77}$$

where r_p is the proton radius.

From (77) we find

$$g^2/\hbar c = (m_p/m_\pi)\nu r_p e^{\nu r_p} \ln(G/e_p) \tag{78}$$

As follows from experiments (Naumov, 1984), in the case under consideration $r_p \approx 1.2$ fm and from (78) and (58), (61) we get

$$g^2/\hbar c \approx 15.6 \tag{79}$$

It is interesting to note that the obtained constant (79) of the high-energy interaction of protons is in accord with its experimental value, which is approximately equal to 15 (Acosta *et al.*, 1973).

Hence the obtained results are in accord with the experimental values of the binding energies of nuclei, their radii, and the constant of strong interaction.

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